

Bell Experiments with Random Destination Sources

Fabio Sciarrino,^{1,2,*} Giuseppe Vallone,^{3,1,*} Adán Cabello,⁴ and Paolo Mataloni^{1,2,*}

¹*Dipartimento di Fisica della “Sapienza” Università di Roma, Roma 00185, Italy*

²*Istituto Nazionale di Ottica (INO-CNR), L.go E. Fermi 6, I-50125 Florence, Italy*

³*Centro Studi e Ricerche “Enrico Fermi”, Via Panisperna 89/A, Compendio del Viminale, Roma 00184, Italy*

⁴*Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*

(Dated: December 10, 2010)

It is generally assumed that sources sending randomly two particles to one or two different observers, named here random destination sources (RDS), cannot be used for genuine quantum nonlocality tests because of the postselection loophole. We demonstrate that Bell experiments not affected by the postselection loophole may be performed with: (i) RDS and local postselection using perfect detectors, (ii) RDS, local postselection, and fair sampling assumption with any detection efficiency, and (iii) RDS and a threshold detection efficiency required to avoid the detection loophole. These results allow the adoption of RDS setups, which are more simple and efficient, for long-distance free-space Bell tests, and extends the range of physical systems which can be used for loophole-free Bell tests.

Introduction.—An experimental loophole-free violation of a Bell inequality is of fundamental importance not only for ruling out the possibility of describing nature with local hidden variable theories [1], but also for proving entanglement-assisted reduction of classical communication complexity [2], device-independent eternally secure communication [3], and random number generation with randomness certified by fundamental physical principles [4]. This explains the interest in loophole-free Bell test over long distances. There are three types of loopholes. The locality loophole [5] occurs when the distance between the local measurements is too small to prevent causal influences between one observer’s measurement choice and the other observer’s result. To avoid this possibility these two events must be spacelike separated. The detection loophole [6] occurs when the overall detection efficiency is below a minimum value, so although the events in which both observer’s have results might violate the Bell inequality, there is still the possibility to make a local hidden variable model which reproduces all the experimental results. To avoid this possibility the overall detection efficiency must be larger than a threshold value. Finally, the postselection loophole [7–9] occurs when the setup does not always prepare the desired state, so the experimenter postselects those events with the required properties. It has been shown that, in certain configurations, the rejection of “undesired” events can be exploited by a *local* model to imitate the predictions of quantum mechanics [7]. However, it has been recently pointed out that the postselection loophole is not due to the rejection of undesired events itself, but rather to the geometry of the setup. The loophole can be fixed with a suitable geometry, without renouncing to the postselection [8, 10]. This leads to the question of when Bell experiments with postselection are legitimate. The answer is interesting since sources with postselection can be simpler and more efficient.

In this paper we analyze schemes which are believed to be affected by the postselection loophole [9, 11, 12]. In particu-

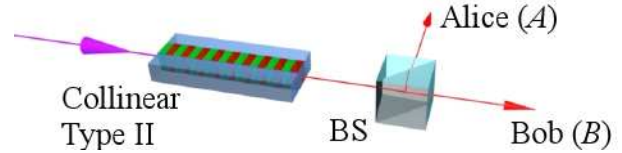


FIG. 1. Collinear source of two photon states. The two particles are produced by type II phase matching and randomly broadcasted to two observers *A* and *B* via a beam splitter (BS).

lar, we focus on random destination sources (RDS) emitting two photons, such that sometimes one photon ends in Alice’s detectors and the other in Bob’s, but sometimes both end in the same party’s detectors. The same argument can be used with massive particles, as discussed later. Hereafter we will consider as benchmark the source shown in Fig. 1 [13]: two photons with horizontal and vertical polarization are generated via collinear spontaneous parametric down conversion (SPDC) and a beamsplitter (BS) splits the photons over the modes *A* and *B*. Three different cases may occur: Both photons emerge on mode *A* ($|HV\rangle_A$), both on mode *B* ($|HV\rangle_B$), or the two photons are divided into different modes $[(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)/\sqrt{2}]$. Such a setup is a natural candidate for long-distance free-space experiments requiring quantum entanglement, and specifically for satellite-based quantum communications [14, 15], since it satisfies the requirements of high efficiency (due to the adoption of periodically poled crystals), stability, compactness (the beam splitter could even be manufactured onto a single chip with the SPDC source), and emission over a single spatial mode.

We will show that Bell experiments not affected by postselection loophole can be performed in these cases: (i) RDS and local postselection with perfect detectors. (ii) RDS, local postselection and fair sampling assumption for any value of detection efficiency. (iii) RDS and a threshold detection efficiency required to avoid even the detection loophole.

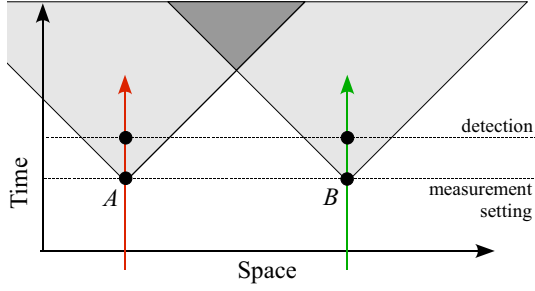


FIG. 2. Space-time diagram for loophole-free Bell experiments.

Let us consider an experiment with RDS producing two photons in different locations, Alice (A) and Bob (B), with probability p , two photons in Alice's side with probability $\frac{1-p}{2}$, and two photons on Bob's side with probability $\frac{1-p}{2}$. Detection efficiencies in Alice and Bob's sides are η_A and η_B , respectively. Alice and Bob also have photon number discriminators. The probability p will depend on the particular configuration. For instance, in the case of the source of Fig. 1, $p = 2T(1 - T)$, where T is the transmittance of the beam splitter.

(i) *Perfect detectors and local postselection.*—Let's consider $\eta_A = \eta_B = 1$. Alice and Bob, depending on the measured number of photons, locally decide to postselect only those events in which entanglement has been successfully distributed. See column $\eta = 1$ of Table I. Is such local discarding of events introducing any loophole?

The answer is no, since the selection and rejection of events is independent of the local measurement settings (otherwise a local hidden variables model could exploit the selection to violate the inequality). Indeed, any Bell experiment with local postselection, in the sense that it does not require Alice and Bob to communicate, is free of the postselection loophole. Local postselection is not a *necessary requirement* to be free of the postselection loophole (see [7] for a counterexample), but is a *sufficient property* to be free of this loophole.

This is a crucial point which deserves a detailed examination. First consider a *selected* event: The two photons have been detected at different locations, corresponding one to Alice's detector D_A and the other to the Bob's detector D_B . If the detection in B is outside the forward light-cone of the measurement setting in A (this is precisely the locality condition), no mechanism could turn a rejected event into a selected one (see Fig. 2). Let us consider a *rejected* event, for example when two photons have been detected at D_A . Both Alice (since she registers a double detection) and Bob (since he does not register any detection) locally discard the event. Again, due to the locality condition, the double detection at Alice's side cannot be caused by Bob's measurement setting, and the absence of Bob detection cannot be influenced by Alice's measurement setting. The same happens

GENERATION			DETECTION					
			$\eta = 1$	$\eta \neq 1$				
	A	B	CHSH (no-fs)	A	B	CHSH (fs)	CH (no-fs)	CHSH (no-fs)
Events I	1	1	✓	1	1	✓	✓	✓
				1	0	NO	✓	✓
				0	1	NO	✓	✓
				0	0	NO	×	✓
Events II	2	0	NO	2	0	NO	×	✓
				1	0	NO	✓	✓
				0	0	NO	×	✓
Events III	0	2	NO	0	2	NO	×	✓
				0	1	NO	✓	✓
				0	0	NO	×	✓

TABLE I. Different types of events (I, II and III), depending on the number of photons sent to observers A and B . Different nonlocality tests are considered according to the detection efficiency (η), the adopted inequality (CH, CHSH) and whether fair-sampling (fs) is assumed. The numbers refers to the photons generated or detected by each observer. NO: Discarded events, ×: Events not discarded but not contributing to the inequality, ✓: Events which contribute to the inequality.

when two photons go towards Bob's side.

Hence, when Alice or Bob locally discard the events, there is no physical mechanism preserving locality which can turn a selected (rejected) event into a rejected (selected) event. The selecting of events is *independent* of the local settings. For the selected events only the result can depend on the local settings. This is exactly the condition under which the Bell's inequalities are valid. Therefore, an experimental violation of them based on local postselection with a random destination source provides a conclusive test of local realism when perfect detectors are used.

(ii) *Fair sampling assumption and local postselection.*—Let's now consider to case of imperfect detection efficiency of the measurement apparatus. Some of the events are lost, and Alice and Bob only keep coincidences and assume fair sampling (*i.e.*, that the coincidences are a statistically fair sample of the pairs in which one photon has gone to Alice and the other to Bob). Under this assumption, Bell experiments based on postselection are able to show violations of Bell's inequalities. Indeed, in the case of $p = 1$, the fair sampling assumption allows Alice and Bob to discard the contribution where just one particle is detected. When $p \neq 1$, we have already shown that the contribution due to double particle detection on the same observer can also be discarded. Hence, when fair sampling is assumed, there is no difference between Bell experiments with or without local postselection.

(iii) *Threshold detector efficiency for loophole-free test with RDS.*—What happens when the fair sampling assumption

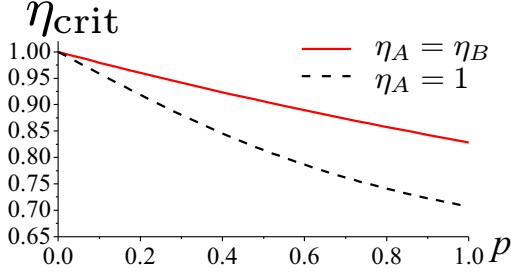


FIG. 3. Critical efficiency for a loophole-free Bell test in the symmetric ($\eta_A = \eta_B > \eta_{\text{crit}}$) or in the asymmetric ($\eta_A = 1$, $\eta_B > \eta_{\text{crit}}$) case.

tion is not considered? We will show that by considering all the possible events, *i.e.*, without postselection or fair sampling, a loophole-free violation with RDS can be obtained with a suitable threshold detection efficiency. For the Clauser-Horne (CH) [16] and the Clauser-Horne-Shimony-Holt (CHSH) Bell inequalities [17], this threshold can be obtained as follows.

In the perfect detection scenario (i), all the events in which two particles are sent to the same observer (events II and III of Table I) are discarded. Here, due to inefficiency, some of the events II and III will contribute to the data and cannot be locally discarded (see the second column of Table I).

Let us consider two observers, Alice and Bob, with dichotomic observables $a_i = \pm 1$ and $b_i = \pm 1$, respectively. Any theory assuming realism and locality must satisfy the CH inequality,

$$I_{\text{CH}} = p(a_1, b_1) + p(a_2, b_1) + p(a_1, b_2) - p(a_2, b_2) - p(a_1) - p(b_1) \leq 0, \quad (1)$$

where $p(a_i, b_j)$ is the probability that Alice obtains $a_i = 1$ and Bob obtains $b_j = 1$, while $p(a_1)$ is the probability that Alice obtains $a_1 = 1$. We adopt one detector in each side corresponding to the +1 outcome. We set $a_i = +1$ ($b_i = +1$) when Alice (Bob) detects only one photon, while $a_i = -1$ ($b_i = -1$) when Alice (Bob) detects zero or two photons. In this way, the inequality is insensitive to any normalization implying that the “vacuum contribution” of standard SPDC sources does not contribute to I_{CH} . Moreover, the events in which two particles are detected by Alice and no particle by Bob (or viceversa), indicated by \times in Table I, do not contribute to Eq. (1). Indeed (1) involves only detection events in which at least one observable is +1.

Let us define Q as the value of I corresponding to the case when both particles are detected, M_A (M_B) the value of I when only particle A (B) is detected from the (1 1) events, T_A (T_B) the value of I when only particle A (B) is detected from (2 0) and (0 2) events, D_A (D_B) the value of I when two particles are detected at side A (B), and X the value when no particle is detected. Then, the average value of I

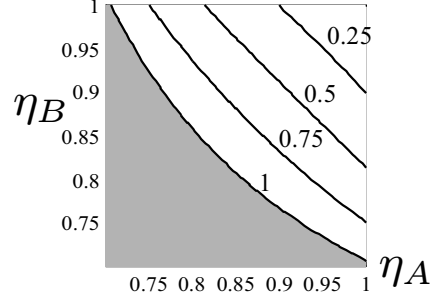


FIG. 4. Allowed values of η_A and η_B for a loophole-free Bell test for different values of p (1, 0.75, 0.5, 0.25). For each value of p the allowed zone is in the upper-right part of the corresponding curve.

will be

$$\begin{aligned} \langle I \rangle = & \eta_A^2 \frac{1-p}{2} (D_A - 2T_A + X) + \eta_A [pM_A + (1-p)T_A - X] \\ & \eta_B^2 \frac{1-p}{2} (D_B - 2T_B + X) + \eta_B [pM_B + (1-p)T_B - X] \\ & + \eta_A \eta_B p(Q - M_A - M_B + X) + X. \end{aligned} \quad (2)$$

It is easy to show that, for the singlet entangled state and choosing the observables a_i and b_i that maximally violate the inequality, we obtain the following values for the CH inequality: $Q = \frac{1}{\sqrt{2}} - \frac{1}{2}$, $M_A = M_B = -\frac{1}{2}$, and $X = 0$. When the two particles are detected by Alice (Bob), we have $a_i = -1$ ($b_i = -1$), which implies $D_A = D_B = 0$. In order to calculate T_A (T_B), it is necessary to know the particular two-photon state sent to Alice (Bob). In most RDS, when two photons are sent to the same observer they have orthogonal polarizations. This implies that when only one photon is detected, we have $T_A = T_B = -\frac{1}{2}$. The local realistic bound is violated when $\langle I \rangle > 0$:

$$\frac{1-p}{2} (\eta_A^2 + \eta_B^2) + p \eta_A \eta_B \left(\frac{1}{2} + \frac{1}{\sqrt{2}} \right) - \frac{1}{2} (\eta_A + \eta_B) > 0. \quad (3)$$

Note that the (0 0), (2 0), and (0 2) events do not contribute to any term in (1).

In the symmetric case ($\eta_A = \eta_B = \eta$), the minimum detection efficiency is

$$\eta > \eta_{\text{crit}} \equiv \frac{2}{2 + p(\sqrt{2} - 1)}. \quad (4)$$

For $p = 1$, we recover $\eta > \frac{2}{1+\sqrt{2}} \simeq 0.83$ [18]. For $p = 0.5$, we obtain $\eta > 0.90$. RDS imposes a stricter constraint on the experimental setting, but still a loophole-free nonlocality test can be achieved.

For the fully asymmetric case ($\eta_A = 1$), we have $\eta_B >$

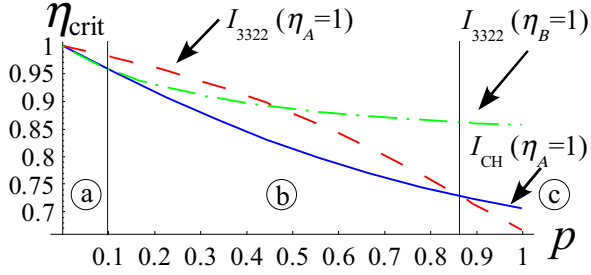


FIG. 5. Critical detection efficiencies in the completely asymmetric case for the CH and the Collins-Gisin inequalities. I_{3322} with $\eta_A = 1$ (dashed line) and with $\eta_B = 1$ (dash-dotted line), CH (Continuous line). Labels (a), (b), and (c) identify the three ranges where different inequalities lead to the lower efficiency threshold.

η_{crit} with

$$\eta_{\text{crit}} = \frac{-1 + p(1 + \sqrt{2}) - \sqrt{4p(1-p) + (1-p - \sqrt{2}p)^2}}{2(p-1)} \quad (5)$$

In the limit $p \rightarrow 1$ we recover $\eta_B > \frac{1}{\sqrt{2}} \simeq 0.71$ [19]. In Fig. 3 we show the critical values of the efficiency in the symmetric and totally asymmetric cases. For the general case (3), Fig. 4 shows the values of η_A and η_B allowing a loophole-free Bell test for different values of p .

It is worth noting that a completely equivalent result is obtained by using the CHSH inequality,

$$I_{\text{CHSH}} = \langle a_1 b_1 \rangle + \langle a_2 b_1 \rangle + \langle a_1 b_2 \rangle - \langle a_2 b_2 \rangle - 2 \leq 0, \quad (6)$$

by using the arguments given in [11]. When one observer detects no particle or two particles, he sets $a_i (b_i) = +1$. When Alice detects two photons and Bob no photon [the (2 0) events], we have $\langle a_1 b_1 \rangle = \langle a_2 b_1 \rangle = \langle a_1 b_2 \rangle = \langle a_2 b_2 \rangle = 1$ [and similarly for the (0 2) events]. The same happens for the (0 0) events (where neither Alice nor Bob detects a particle). If the source produces the singlet state when the two particles are sent to different observers, inequality (3) holds.

Finally, we demonstrate that, in the asymmetric case $\eta_A \neq \eta_B$, an inequality with lower bound with respect to (5) does exist for some values of p . Consider the I_{3322} inequality [20],

$$\begin{aligned} I_{3322} = & p(a_1, b_1) + p(a_1, b_2) + p(a_1, b_3) + p(a_2, b_1) \\ & + p(a_2, b_2) + p(a_3, b_1) - p(a_2, b_3) - p(a_3, b_2) \\ & - 2p(a_1) - p(a_2) - p(b_1) \leq 0. \end{aligned} \quad (7)$$

By setting $a_i = 1$ ($b_i = 1$) when Alice (Bob) detects zero or two photons, it is possible to show that the inequality is violated if

$$\frac{1-p}{2}(\eta_A^2 + 3\eta_B^2) + p\eta_A\eta_B\frac{9}{4} - \frac{1}{2}(\eta_A + 3\eta_B) > 0. \quad (8)$$

Eq. (8) depends in a different way on η_A and η_B , hence we will consider separately the two conditions $\eta_A = 1$ and $\eta_B = 1$. We may compare the efficiency threshold in Fig. 5. The plot is divided in three regions (a–c), depending on which inequality leads to the lowest efficiency threshold.

For $\eta_A = 1$, the lower bound on η_B is $\eta_B > 4p/(9p - 6 + \sqrt{36 - 60p + 33p^2})$, which is better than (5) for any $p > 0.863$ (c). For $p = 1$, we obtain the same results presented in [21]. By setting $a_i = -1$ ($b_i = -1$), when Alice (Bob) detects zero or two photons, we obtain the same result with $\eta_A \leftrightarrow \eta_B$. In this case, for $\eta_A = 1$, the lower bound on η_B is better than the CH condition for any $p < 0.099$ (see Fig. 5) (a). In the central region (b), CH is still the optimal choice. Due to the T_A and T_B terms in (2), the efficiency bound depends on the specific form of the source. Here we have calculated the bound for the case of two photons sent to the same observers with orthogonal polarization.

Finally, RDS are useful for loophole-free Bell tests beyond the most promising proposals to date (see [22] and references therein). The idea is to combine RDS of pairs of massive particles such as neutrons or molecules with interferometric setups like [8] into Bell experiments with postselection, and take advantage of the fact that the detection efficiencies for these particles are above the thresholds obtained in this paper. So far, the scheme in [8] has been tested with photons [10] and electronic currents [23], but there seems to be no fundamental problem in performing similar experiments with molecules (see [24] for an example of a RDS with molecules) and accelerator-based sources of neutrons [25].

This work was supported by FIRB Futuro in Ricerca - HYTEQ. A.C. acknowledges support from the Spanish MCI Project No. FIS2008-05596.

* <http://quantumoptics.phys.uniroma1.it>

- [1] J. S. Bell, *Physics* **1**, 195 (1964).
- [2] H. Buhrman, R. Cleve, S. Massar, and R. de Wolf, *Rev. Mod. Phys.* **82**, 665 (2010).
- [3] A. Acín *et al.*, *Phys. Rev. Lett.* **98**, 230501 (2007).
- [4] S. Pironio *et al.*, *Nature (London)* **464**, 1021 (2010).
- [5] J. Bell, *J. Phys. C* **2**, 41 (1981).
- [6] P. M. Pearle, *Phys. Rev. D* **2**, 1418 (1970).
- [7] S. Aerts, P. Kwiat, J.-Å. Larsson, and M. Żukowski, *Phys. Rev. Lett.* **83**, 2872 (1999).
- [8] A. Cabello *et al.*, *Phys. Rev. Lett.* **102**, 040401 (2009).
- [9] L. De Caro and A. Garuccio, *Phys. Rev. A* **50**, R2803 (1994).
- [10] G. Lima *et al.*, *Phys. Rev. A* **81**, 040101(R) (2010).
- [11] S. Popescu, L. Hardy, and M. Żukowski, *Phys. Rev. A* **56**, R4353 (1997).
- [12] P. G. Kwiat, P. H. Eberhard, A. M. Steinberg, and R. Y. Chiao, *Phys. Rev. A* **49**, 3209 (1994).
- [13] A. Martin *et al.*, *Opt. Express* **17**, 1033 (2009).
- [14] R. Ursin *et al.*, *IAC Proceedings* **A2.1.3** (2008), quant-ph/0806.0945.

- [15] X.-M. Jin *et al.*, Nat. Photon. **4**, 376 (2010).
- [16] J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).
- [17] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
- [18] A. Garg and N. D. Mermin, Phys. Rev. D **35**, 3831 (1987).
- [19] A. Cabello and J.-Å. Larsson, Phys. Rev. Lett. **98**, 220402 (2007).
- [20] D. Collins and N. Gisin, J. Phys. A **37**, 1775 (2004).
- [21] N. Brunner, N. Gisin, V. Scarani, and C. Simon, Phys. Rev. Lett. **98**, 220403 (2007).
- [22] W. Rosenfeld *et al.*, Adv. Sci. Lett. **2**, 469 (2009).
- [23] D. Frustaglia and A. Cabello, Phys. Rev. B **80**, 201312(R) (2009).
- [24] C. Gneiting and K. Hornberger, Phys. Rev. Lett. **101**, 260503 (2008).
- [25] Y. Hasegawa (private communication).